**DISCLAIMER**

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The impedance and response of a new beam position monitor (BPM) geometry have been calculated using the POISSON code\(^1\) implemented on a personal computer. A vertical BPM consists of upper and lower electrodes, on which voltages, \( V_1 \) and \( V_2 \), respectively, are induced by the electron bunches. The BPM electrodes are contained within a beam pipe with inner radius of 1-3/8" (3.4925 cm); the dimensions as constructed\(^2\) are shown in Figure 1.

Impedance was calculated by electrostatic modeling of a single electrode in the grounded beam pipe as a constant potential surface extending indefinitely in the longitudinal direction. With an electrode voltage, \( V_e \), of 1000 volts, POISSON was used to calculate the stored electrostatic energy per unit length, \( E/I \). The TEM waveguide mode has characteristic impedance, \( Z_e \), which is related to the electrostatic stored energy by: \( Z_e = V_e^2/2c(E/I) \), where \( "c" \) is the speed of light.

As a test, we successfully modeled a 50 ohm coaxial transmission line whose outer conductor radius is 2.3 times the inner conductor radius. For the BPM geometry of Fig. 1, a characteristic impedance of 46.46 ohms was calculated. To investigate the perturbation from the second electrode, we included the second electrode as either an additional region at ground potential, or a floating electrode (simulated as a dielectric with a large relative permittivity of 10000). The calculated impedances of these cases were 46.44 and 46.45 ohms, indicating an insignificant perturbation.

The calculated electrode impedance was increased to 50 ohms by shaving 0.0974 cm from the left and right sides of the electrodes, as shown in Fig. 2. The electrode geometry of Fig. 2 was used in subsequent calculations.

The response of the BPM to the electron beam position was obtained by simulating the electrostatic voltages induced on the floating electrodes by an imaginary beam, consisting of a constant 1000 volt potential surface of 1 mm radius centered on the beam position \((x_0, y_0)\). The floating electrodes were each modeled as a dielectric with a large relative permittivity of 10,000. With such high permittivity, the voltage variation within each electrode was only \( \sim 0.02 \) volt for a 2 cm beam height. The induced voltages in the center of the upper and lower electrodes, \( V_1 \) and \( V_2 \), were determined by POISSON. The quantity, \((V_1-V_2)/(V_1+V_2)\), is the signal response to the beam position, and is independent of the beam current. In Fig. 3, the signal response is plotted versus beam height for three different horizontal beam locations. According to POISSON, the signal \((V_1-V_2)/(V_1+V_2)\) is linearly related to the vertical beam position, \( y_o \), within 5% for \( bx_o, by_o < 0.5 \) cm, and within 10% for \( bx_o, by_o < 1 \) cm. The linear response, \((V_1-V_2)/(V_1+V_2)/y_o\), is 0.58 cm\(^{-1}\). The reciprocal linear response is 1.72 cm.
The precision of POISSON calculations was tested by studying mesh sizes different from that used above (dx = dy = 0.05 cm), and relative dielectric different from 10,000. For mesh sizes of 0.02 ≤ dx = dy ≤ 0.05 cm, and relative dielectric 500 ≤ ε/ε₀ ≤ 20,000, the results agreed within ~ 0.1%.

We also studied the difference in BPM response between electrodes with the original dimensions of Fig. 1, and the electrodes of Fig. 2, obtained by shaving 0.0974 cm from the right and left sides. The responses differed by an insignificant 0.3%, which is comparable to the accuracy of the calculations.

In summary, the characteristic impedance of the constructed electrode geometry of Fig. 1 was calculated by POISSON to be 46.5 ohms. Shaving 0.0974 cm from the right and left sides of the electrodes gives the geometry of Fig. 2, for which the characteristic impedance is 50 ohms. These BPM designs give a signal, \((V₁-V₂)/(V₁+V₂)\), which is linearly proportional to the beam height, \(y_b\), within 5% for beam locations with \(lx_J, ly_J < 0.5 \text{ cm}\). The BPM signal is within 10% of the linear response for beam locations with \(lx_J, ly_J < 1 \text{ cm}\). The linear response, \((V₁-V₂)/(V₁+V₂))/y_b\), is 0.58 cm⁻¹. The reciprocal linear response is 1.72 cm⁻¹.

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Fig 1: BPM as constructed. Coordinates in cm.
Fig. 2 - BPM dimensions with electrodes shaved so that $Z_c = 50$ ohms.
Figure 3
Appendix: Poisson code input files:

(a) Automesh input - 8 BPM electrodes as constructed
(b) Automesh input - electrodes shaved so that Zc = 50 ohms.
(c) Poisson input for \( \varepsilon / \varepsilon_0 = 10.0 \).