DISCLAIMER

SRC Technical Notes are informal memos intended for internal communication and documentation of work in progress. These notes are not necessarily definitive and have not undergone a pre-publication review. If you rely on this note for purposes other than its intended use, you assume all risk associated with such use.
Extraction of edge radiation within a straight section of an electron storage ring

R. A. Bosch

Synchrotron Radiation Center, University of Wisconsin-Madison
3731 Schneider Dr., Stoughton, WI 53589

Abstract. In order to increase the long wavelength range of an edge radiation beamline, radiation from the bending magnet edge at the beginning of a straight section may be collected by a mirror within the straight section. This method avoids the suppression of long wavelengths downstream of a straight section from the large opening angles of near-field edge radiation and destructive interference of the edge radiation sources at the beginning and end of the straight section.
Extraction of edge radiation within a straight section of an electron storage ring

R. A. Bosch

Synchrotron Radiation Center, University of Wisconsin-Madison
3731 Schneider Dr., Stoughton, WI 53589

Abstract. In order to increase the long wavelength range of an edge radiation beamline, radiation from the bending magnet edge at the beginning of a straight section may be collected by a mirror within the straight section. This method avoids the suppression of long wavelengths downstream of a straight section from the large opening angles of near-field edge radiation and destructive interference of the edge radiation sources at the beginning and end of the straight section.

I. INTRODUCTION

The radiation from ultrarelativistic electrons entering or exiting a bending magnet (edge radiation (1,2,3,4)) may be used as a bright infrared (IR) source. When edge radiation is collected downstream of a straight section, an aperture is expected to intercept wavelengths, \( \lambda_e \), exceeding \( \lambda_{ap} = \theta_{ap} R_{ap} \), where \( \theta_{ap} \gg 1/\gamma \) is the angular extent of the aperture and \( R_{ap} \) is the distance from the bending magnet edge to the aperture (5). In addition, destructive interference between the edge radiation sources at the beginning and end of the straight section is expected to suppress far-field radiation at wavelengths exceeding 10–100 \( L \gamma^2 \), where \( L \) is the straight section length, and \( \gamma \) is the relativistic mass factor (6). Thus, the edge radiation downstream of a straight section may be limited to \( \lambda < \min(100 \ L \gamma^2, \lambda_{ap}) \).

For example, the edge radiation beamline operated at the 800 MeV (\( \gamma = 1567 \)) electron storage ring, Aladdin, has an aperture with \( \theta_{ap} = 4.5 \) mrad at \( R_{ap} = 1.4 \) m., so that \( \lambda_{ap} = 28 \) \( \mu \)m. The straight section length, \( L = 3 \) m, gives 100 \( L \gamma^2 = 120 \) \( \mu \)m. Consequently, the flux is expected to roll off at wavelengths exceeding 28 \( \mu \)m.

A computation of edge radiation within the straight section indicates that these mechanisms of long-wavelength suppression are absent, so that the long wavelength range should be extended. Consequently, this method of extracting edge radiation may be useful in constructing a far infrared beamline.

II. THE ACCELERATION FIELD

Consider the radiation within a straight section of an electron storage ring. At a location upstream of the straight section end, radiation from the bending magnet at the downstream end of the straight section may be neglected. In SI units, the "acceleration" field, \( \vec{E}_a \), of an electron which exits the bending magnet at the beginning of the straight section at time, \( t = 0 \), obeys (7):

\[
\vec{E}_a(\vec{x},\omega) = \int_{-\infty}^{\infty} \vec{E}_a(\vec{x},t) e^{i\omega t} dt = \frac{e}{4\pi\varepsilon_0 c R} \int_{t_1}^{0} \left[ \hat{n} \times \left( \hat{n} \times \frac{\vec{B}}{\gamma \sqrt{1 - \frac{\hat{n} \cdot \vec{B}}{\gamma}} \right) \right] e^{i\omega t} dt,
\]

Here, \( \vec{x} \) is the observer location, \( \omega \) is angular frequency, \( c \) is the speed of light, \( \vec{B}(t) \) is the electron velocity divided by \( c \), \( e < 0 \) is the electron charge, \( \varepsilon_0 \) is the permittivity of free space, \( \vec{R} \) is the distance from the electron to the observer, and \( \hat{n}(t) \) is the unit vector pointing from the electron location to the observer. The time, \( t_1 < 0 \), is sufficiently small that the electron has yet to be deflected by an angle \( \gg 1/\gamma \), so that radiation emitted before this time is not directed along the straight-section axis.
For a rapid deflection in a bending magnet exit located at the origin, \( R = |\tilde{x}| \) and \( \hat{n} = \tilde{x} \perp R \) are constant in time, while the observation time, \( t_e \), is related to the time, \( t \), of radiation emission by

\[
\frac{dt_e}{dt} = 1 - \hat{n} \cdot \hat{\beta}(t) .
\]  

(2)

For observation angles (measured relative to the straight section axis) \( \theta \sim 1/\gamma \), the variation in the phase term of eq. (1), \( \phi(t_e) - \phi(0) \sim (2\pi/c\lambda)(1/2\gamma^2 + \theta^2/2)\gamma \), is small compared to \( \pi \) provided that

\[
\lambda \gg \gamma d^2 \gamma ,
\]  

(3)

where \( d = \beta c t_e \) the “edge length,” is the distance traversed while being deflected through an angle \( 1/\gamma \). For wavelengths obeying eq. (3), we consider the zero edge length (\( d \to 0 \)) model in which \( \exp(\imath \omega \phi(t)) \) is considered constant in eq. (1).

Integrating an exact differential yields, for \( t_e(0) = 0 \),

\[
\vec{E}_e(\tilde{x}, \omega) = \frac{e}{4\pi \varepsilon_0 c R} \frac{\hat{n} \times (\hat{n} \times \hat{\beta})}{1 - \hat{n} \cdot \hat{\beta}} \bigg|_{t_e}^0 .
\]  

(4)

The RHS of eq. (4) evaluated at time \( t = t_e \) is negligible for observer locations with \( \theta \sim 1/\gamma \), by definition of \( t_e \) so that

\[
\vec{E}_e(\tilde{x}, \omega) = \frac{e}{4\pi \varepsilon_0 c R} \frac{\hat{n} \times (\hat{n} \times \hat{\beta})}{1 - \hat{n} \cdot \hat{\beta}} \bigg|_0 .
\]  

(5)

The radiation described by eq. (5) is radially polarized and independent of wavelength, forming a cylindrically-symmetric hollow cone. The radial electric field is approximately

\[
E_r(\tilde{x}, \omega) = \frac{e\gamma}{2\pi \varepsilon_0 c R} \frac{\varphi}{2(1 + \varphi^2)} ,
\]  

(6)

where \( \varphi = \gamma \theta \) is the normalized observation angle. The peak field occurs when \( \varphi = 1 \), i.e., \( \theta_{\text{opening}} - \theta_{\text{peak}} = 1/\gamma \).

III. THE VELOCITY FIELD

The total radiation field is the sum of the “acceleration field” and the “velocity field.” The acceleration field dominates in the far field. However, the velocity field must be considered for locations near to the electron orbit. Because negligible radiation is received from an electron downstream of the observer, we neglect the downstream bending magnet, and consider an electron which travels an infinitely-long straight path after suddenly exiting a bending magnet at \( t = 0 \). In this case, the velocity field, \( \vec{E}_v \), is (7)

\[
\vec{E}_v(\tilde{x}, \omega) = \int_{-\infty}^{\infty} \vec{E}_v(\tilde{x}, t_e) e^{i\omega \phi(t_e)} dt_e = \frac{e}{4\pi \varepsilon_0 c R^2} \int_0^\infty \frac{\hat{n}(t) - \hat{\beta}}{(1 - \hat{n}(t) \cdot \hat{\beta})^2 R(t)^2} e^{i\omega \phi(t)} dt .
\]  

(7)

Here, \( \hat{n}(t) \) is the unit vector directed from the electron to the observer, and \( R(t) \) is the distance from the electron to the observer. The contribution from \( t < 0 \) is insignificant along the straight section axis because of the large value of \( 1 - \hat{n}(t) \cdot \hat{\beta} \) for \( t < 0 \). Because \( \int_0^\infty dt = \int_0^\infty dt - \int_{-\infty}^0 dt \), the velocity field may be written as the difference of two fields.
\[ E_{s}(\vec{x}, \omega) = E_{\text{line}}(\vec{x}, \omega) - E_{s, \text{entrance}}(\vec{x}, \omega) \]

where

\[ E_{\text{line}}(\vec{x}, \omega) = \frac{e}{4\pi e_{0} c^{2}} \int_{-\infty}^{\infty} \frac{\vec{n}(t) - \vec{B}}{(1 - \vec{n}(t) \cdot \vec{B})^{1.5}} R(t^{'}) e^{i\omega t} dt \]

and

\[ E_{s, \text{entrance}}(\vec{x}, \omega) = \frac{e}{4\pi e_{0} c^{2}} \int_{0}^{\infty} \frac{\vec{n}(t) - \vec{B}}{(1 - \vec{n}(t) \cdot \vec{B})^{1.5}} R(t^{'}) e^{i\omega t} dt \]

Both fields describe cylindrically-symmetric, radially-polarized, radiation. The field, \( E_{\text{line}} \), is that produced by an electron traveling in a straight line for all time. This field is the Lorentz transform of the static Coulomb field. In the time-domain and SI units, the Lorentz transform of the Coulomb field is (7)

\[ E_{\text{line}}(\vec{x}, t^{'}) = \frac{\gamma e r}{4\pi e_{0} \left( r^{2} + \gamma^{2} \beta^{2} c^{2} t^{2} \right)^{3/2}} \]

where \( r \) is the distance of the observer from the straight line. The time coordinate in eq. (11), \( t' \), is zero at the time of closest approach, and related to the time coordinate, \( t_e \), by \( t_e = t + (R/2\gamma^{2} c)(1-\beta^{2}) \), where the observation time, \( t_e \), is defined to be zero when the radiation from the bending magnet edge reaches the observer. In the frequency domain, eq. (11) becomes

\[ E_{\text{line}}(\vec{x}, \omega) = \frac{e\gamma f}{2\pi e_{0} c R} \left( \frac{e^{i\pi R_{e}(1-\beta^{2})}}{2\pi} \right) \int_{0}^{\infty} \frac{\cos(2\pi R_{e} \varphi \omega)}{(1 + \omega^{2})^{1/2}} \varphi d\varphi \]

where \( R_{e} = R/\lambda \) is the normalized observer distance for wavelength, \( \lambda \), and the exponential phase term is \( \exp(i\omega t_e - t') \). At the wavelength, \( \lambda \), the field, \( E_{\text{line}} \), is proportional to \( 1/r \) for \( r \ll \lambda \gamma \), and dies off rapidly at larger radii.

The field, \( E_{s, \text{entrance}} \), is equivalent to the velocity field produced by an electron entering a bending magnet at time, \( t = 0 \), after traversing an infinitely-long straight section. For observer angles, \( \beta \ll 1 \) rad, the radial field is given by (5)

\[ E_{s, \text{entrance}}(\vec{x}, \omega) = \frac{e\gamma f}{2\pi e_{0} c R} \left( \frac{e^{i\pi R_{e}(1-\beta^{2})}}{\varphi} \right) \int_{0}^{\infty} \frac{w^{1/2} e^{i\pi R_{e}(w-1/w)}}{w^{1/2}} d\varphi \]

The velocity field is given by the difference of the RHS of eqs. (12) and (13). The total field from a single electron, \( E(\vec{x}, \omega) \), is given by adding the acceleration field (the RHS of eq. (6)) to the velocity field. For an electron bunch length exceeding the observed radiation wavelength, the photon flux per unit solid angle (in photons/s-relative bandwidth \( \Delta\nu/\nu \)-steradian) from a current, \( I \), is given by (7)

\[ \frac{dF}{d\Omega} = \frac{\Delta\nu I}{\omega} \left( \frac{2eR_{e} \gamma}{\epsilon} \right)^{2} |E(\vec{x}, \omega)|^{2} \]

where \( \alpha = 1/137 \) is the fine-structure constant.
IV. THE TOTAL FIELD

The integrals in eqs. (12) and (13) were evaluated numerically to determine the velocity field and the total field generated by a single electron according to the zero edge length model. The velocity field and total field depend on the normalized wavelength, \( \lambda \gamma^2 R \), and the observation angle, \( \theta \), while the acceleration field depends only on the observation angle. Figure 1 displays \( |E| \gamma^2 \) at the wavelength, \( \lambda = R \gamma^2 \), where \( R \) is the observer distance from the bending magnet edge. For \( \gamma \theta < 1/2 \), the

FIGURE 1. Upstream edge radiation for \( \lambda = R \gamma^2 \).

FIGURE 2. Upstream edge radiation for \( \lambda = 100 \ R \gamma^2 \).
total field is approximated by the velocity field, while for $\gamma \theta > 1/2$, the total field is approximated by the acceleration field.

For short wavelengths with $\lambda < R\gamma^2$, the velocity field is nearly equal to that of an infinitely-long line, $E_{\text{line}}$, which dies off for $r > \lambda \gamma$, i.e. $\theta > \lambda \gamma R$. Thus, the total field is nearly equal to the acceleration field for $\gamma \theta > \lambda \gamma^2 R$.

For long wavelengths with $\lambda >> R\gamma^2$, the velocity field and total field become independent of wavelength, with negligible wavelength dependence for $\lambda > 10 R\gamma^2$. Figure 2 displays numerical computations for $\lambda = 100 R\gamma^2$. The total field is approximated by the velocity field for $\gamma \theta < 1$; the velocity field dies off for larger values of $\theta$. For $\gamma \theta > 2$, the total field is approximated by the acceleration field.

Thus, in the zero edge length model, for $\theta > 2\gamma$, the total field is approximated by the acceleration field (i.e., the far-field radiation distribution) for all wavelengths. For small angles where $\theta < \min(\lambda \gamma R, 1/\gamma)$, the Lorentz transform of the Coulomb field, $E_{\text{line}}$, dominates. Because $E_{\text{line}} \approx 1/\theta$, the flux density $d\Phi/d\Omega = 1/\theta^2$, so that the integrated flux over angles, $\int_{\theta_{\text{min}}}^{\theta < 2\gamma} d\Phi/d\Omega$, diverges logarithmically as $\theta_{\text{min}} \to 0$.

In comparison with the far-field edge radiation downstream of a straight section, there is an increase in flux at $\theta < \min(\lambda \gamma R, 1/\gamma)$, where the field, $E_{\text{line}}$, contributes large flux densities in the center of the edge radiation angular distribution. For $\theta > 2\gamma$, the flux is approximated by the far-field distribution from a single edge. Because the radiation distribution does not broaden at long wavelengths from near-field effects, extraction of edge radiation within a straight section allows the possibility of extended long wavelength range.

V. FINITE EDGE LENGTH

The above results from the zero edge length model are expected to apply for sufficiently-long wavelengths that the requirement, $\lambda >> R\gamma^2$, is satisfied. To examine the effects of finite edge length on the infrared radiation at the 800 MeV electron storage ring, Aladdin, we modeled a linear fringe field, ramping down over a distance of 10.8 cm from a uniform-field region in which the radius of curvature is 2.083 m (3). For this case, $d\tau = 7$ mm (6). For a ring current of 200 mA, we consider an observer 3 m downstream of the beginning of a straight section. Figure 3 shows the flux densities computed for a 10.8 cm linear fringe field and for the zero edge length model. For $\lambda \geq 10$ mm, the zero edge length model provides a good approximation, as in the cases of near-field (5) and far-field (6) radiation downstream of a straight section.

![Figure 3](image_url)

**FIGURE 3.** Flux density from numerical computations (solid lines) and the zero edge length model (dashed lines) for an observer 3 m downstream of the beginning of an Aladdin straight section with an electron current of 200 mA and energy of 800 MeV. The flux is plotted versus horizontal position in the plane of the electron orbit, for electrons deflected by the bending magnet in the negative x-direction. (a) $\lambda = 1$ mm. (b) $\lambda = 10$ mm. (c) $\lambda = 100$ mm. (d) $\lambda = 1000$ mm.
VI. EXTRACTION OF RADIATION

We consider a mirror tilted at 45 degrees to extract edge radiation from an Aladdin upstream bending magnet, at a distance 3 m downstream of the bending magnet edge. The undulator vacuum sections have already established a vertical aperture of $|y| = 9.5$ mm, so the beam lifetime should not be affected by a two-part mirror which collects over the region, $|y| > 9.5$ mm, inside the beam pipe radius of 32 mm. For 800 MeV operation ($\gamma = 1567$), this collection region corresponds to vertical observation angles, $|\theta| > 3.17$ mrad = 4.96$m^\circ$, and $\theta < 10.66$ mrad = 16.71$m^\circ$. Because $\theta > 2\gamma$ over the collection region, the far-field edge radiation distribution (given by the acceleration field) may be used to evaluate the collected flux for $\lambda \geq 10$ $\mu$m, with the result:

$$Flux (R = 3 \text{ m}) = 1.41 \alpha (\Delta \omega / \omega) (I / \epsilon \tau)$$

(15)

where $\alpha = 1/137$ is the fine structure constant, $\Delta \omega / \omega$ is the relative bandwidth, and $I$ is the current. This flux is nearly equal to the far-field edge radiation flux in the central bright spot (4,5). $F(\theta < 3 / \gamma) = 1.40 \alpha (\Delta \omega / \omega) (I / \epsilon \tau)$.

Thus, the collected flux is expected to be comparable to that of the existing edge radiation beamline downstream of a straight section, without the expected long wavelength suppression at $\lambda > 28$ $\mu$m. The absence of edge radiation at short wavelengths ($\lambda < d \gamma / \gamma = 7$ $\mu$m) should reduce the heat load on the mirror. Furthermore, collection of IR radiation within a straight section will not reduce the number of bending magnet beamlines which may be constructed on Aladdin.

VII. SUMMARY

The edge radiation was considered at a location upstream of the downstream end of a straight section. At such a location, the edge radiation at angles $\theta$, exceeding $2\gamma$, is approximated by the acceleration field (i.e., the far-field radiation distribution). This differs from the near-field edge radiation downstream of a straight section, where the velocity field cancels the acceleration field for $\theta << \sqrt{\gamma / \gamma}$ when $\lambda > R \gamma$, resulting in an angular broadening at long wavelengths. Consequently, the long-wavelength range of edge radiation may be extended by extraction of edge radiation within a straight section.

For the Aladdin electron storage ring, extraction within a straight section is expected to produce comparable flux to that contained in the central bright spot downstream of a straight section, without the suppression at long wavelengths ($> 28$ $\mu$m) expected downstream of a straight section. Consequently, this extraction method may be useful in constructing a far infrared beamline.

ACKNOWLEDGMENTS

The author appreciates valuable discussions with O. V. Chubar, M. A. Green, T. E. May, R. Reiningher and W. S. Trzeciak. This work was supported by NSF grant DMR-95-31009.

REFERENCES