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University of Wisconsin-Synchrotron Radiation Center **TECHNICAL NOTE**	**File No.** **SRC-217**	**Page** 1 of 8
Subject: **Longitudinal wake of a suddenly accelerated electron bunch**	**Author(s):** **R. A. Bosch**	
	Date: **February 23, 2007**	

We consider the longitudinal wake of an electron bunch that is suddenly accelerated. This wake approximates the edge-radiation wake of an electron exiting a bending magnet, the wake of an electron accelerated in a high-field gradient, and the wake of forward transition radiation. The on-axis wake is large within the radiation formation zone, where it provides resistive impedance that decelerates the bunch electrons. A comparison with the computed wake downstream of a bending magnet yields good agreement. For schemes in which a bunch produced by laser-plasma acceleration drives a VUV or xray FEL, the wake causes large energy losses that may spoil the FEL process.

1. Introduction

The longitudinal wake of an electron bunch accelerated through a high field gradient may have a substantial effect upon the energy of the electrons in the bunch. The variation of electron energies within the bunch, as well as the bunch's energy loss when propagating though an undulator, may influence the operation of a free electron laser (FEL) driven by the bunch [1]. For an ultrarelativistic bunch, evaluation of this effect may require the knowledge of the longitudinal wake within its formation zone [1].

Starting with an expression for the longitudinal electric field of a bunch that is suddenly deflected or decelerated [2], we obtain the wake of a bunch that is suddenly accelerated. This wake also describes forward transition radiation. The wake approximates the wake of a bunch exiting a bending magnet at distances from the magnet edge exceeding the formation length of the magnet's coherent synchrotron radiation.

A comparison with the wake downstream of a bending magnet yields good agreement. We also calculate the wake for schemes in which a bunch produced by laser-plasma acceleration drives a VUV or xray FEL. In these cases, the wake causes large energy losses that may spoil the FEL process.

2. Suddenly stopped electron

When an electron is suddenly stopped or deflected, the longitudinal electric field is given by the sum of the Liénard-Wiechert acceleration and velocity fields. In paraxial approximation and SI units, the longitudinal acceleration field downstream of an electron that travels along the z-axis until it is deflected at the origin at time t = 0 is given in the frequency domain by [2]

$$\tilde{E}_a(R,\theta,\omega) = \int_{-\infty}^{\infty} E_a(R,\theta,t)e^{i\omega t}dt = e^{ikR}\,\frac{e}{2\pi\varepsilon_0 cR}\left[\frac{(\gamma\theta)^2}{1+(\gamma\theta)^2}\right] \quad . \tag{1}$$

Here, R is the distance from the origin, $\theta << \pi/2$ is the observation angle with respect to the z-axis, ω is angular frequency, $e < 0$ is the electron charge, c is the speed of light, ε_0 is the permittivity of free space, γ is the relativistic factor, and $k = \omega/c$ is the wavenumber. The wavelength is $\lambda = 2\pi c/\omega$. The longitudinal velocity field is [2]

$$\tilde{E}_v(R,\theta,\omega) = e^{ikR}\,\frac{e}{2\pi\varepsilon_0 cR}\,\frac{e^{i\pi R_n\left(1-\varphi^2\right)}}{\varphi}\int_0^{\varphi}\frac{1-x^2}{\left(1+x^2\right)^2}e^{i\pi R_n\varphi(x-1/x)}dx \quad , \tag{2}$$

where $\varphi \equiv \gamma\theta$ and $R_n \equiv R/\lambda\gamma^2$. Outside the formation zone ($R >> \lambda\gamma^2/2\pi$), the velocity field vanishes. Within the formation zone ($R << \lambda\gamma^2/2\pi$), the velocity field is approximately [2]

$$\tilde{E}_v(R,\theta,\omega) = e^{ikR}\,\frac{e}{2\pi\varepsilon_0 cR}\left[\frac{1}{1+(\gamma\theta)^2}\right] \quad . \tag{3}$$

Summing the acceleration and velocity fields gives the longitudinal field within the formation zone [2]

$$\tilde{E}(R,\theta,\omega) = e^{ikR}\,\frac{e}{2\pi\varepsilon_0 cR} \quad . \tag{4}$$

Outside the formation zone, the on-axis longitudinal field, given by eq. (1), is zero. Consequently, the on-axis longitudinal field of a suddenly decelerated bunch is largest within the formation zone, where it will accelerate any electrons that are present.

The above result applies in free space. When an electron travels parallel to a conducting plate, located at a distance $h/2$ from the z-axis, the on-axis field given by eq. 4 [$e^{ikz}(e/2\pi\varepsilon_0 cz)$] is supplemented by that of an opposite image charge traveling at a distance h from the axis. For $h << z$, the on-axis field from the image charge is approximately $e^{ikz}e^{ikh^2/2z}(-e/2\pi\varepsilon_0 cz)$, which cancels the field of eq. (4) for $kh^2/2z << \pi$. This cancellation — shielding of the longitudinal field by a wall of a vacuum chamber, therefore occurs for $z >> h^2/\lambda$, where h is the height of the chamber. This is the same region where the transverse field is shielded [3]. For smaller z, the image charge produces an oscillating field upon a charge traveling on-axis with ultrarelativistic velocity. Therefore, in a vacuum chamber of height h, eq. (4) may be used to describe the non-oscillatory portion of the longitudinal wake for $z << \min(\lambda\gamma^2/2\pi, h^2/\lambda)$.

In this note, we study analytically the approximate longitudinal field given by eq. (4) for $z < \min(\lambda\gamma^2/2\pi, h^2/\lambda)$ and zero for $z > \min(\lambda\gamma^2/2\pi, h^2/\lambda)$. More exact results may be obtained by using eq. (2) to describe the velocity field of a suddenly stopped electron, using image charges to represent the vacuum chamber, or by numerical solution of the Liénard-Wiechert fields that describe a gradual deceleration.

3. Suddenly accelerated electron

The radiation field downstream of a suddenly accelerated electron is given by subtracting the field of a suddenly stopped electron from the Coulomb field of an electron in uniform motion. For an electron traveling on the z-axis, which passes through the origin at time $t = 0$, the longitudinal Coulomb field expressed in cylindrical coordinates $r = R\theta$ and z is [4]

$$\tilde{E}_{Coul}(R,\theta,\omega) = e^{ikz}\exp(ikz/2\gamma^2)\left(\frac{-ie}{\varepsilon_0 c\lambda\gamma^2}\right)K_0\left(\frac{\omega r}{c\gamma}\right) \quad (5)$$

Within the formation zone, subtracting eq. (4) from eq. (5) gives the total longitudinal field of a suddenly accelerated electron

$$\tilde{E}(r,z,\omega) = e^{ikz}\left[\left(\frac{-ie\omega}{2\pi\varepsilon_0 c^2\gamma^2}\right)K_0\left(\frac{\omega r}{c\gamma}\right) - \left(\frac{e}{2\pi\varepsilon_0 c}\right)\frac{\exp\left(ik\sqrt{z^2+r^2}-ikz\right)}{\sqrt{z^2+r^2}}\right] \quad (6)$$

4. Wake of a bunch

Consider a cylindrically symmetric bunch of N electrons whose current density through the $z = 0$ plane at radius r and time t is $Ne\rho(r)f(t)$, where $2\pi\int_0^\infty r dr\rho(r) = \int_{-\infty}^\infty f(t)dt = 1$. Let the form factor $F(\omega)$ equal $\int_{-\infty}^\infty f(t)\exp(i\omega t)dt$. For a Gaussian bunch, $F(\omega) > 0.6$ for $\lambda > 2\pi\sigma_z$, where $\sigma_z = c\sigma_t$ is the longitudinal bunch length. The bunch's radiation formation zone is approximated by the zone for $\lambda = 2\pi\sigma_z$, whose formation length is $(2\pi\sigma_z)\gamma^2/2\pi = \sigma_z\gamma^2$.

The longitudinal electric field of the bunch in the frequency domain is

$$\tilde{E}_N(r,z,\omega) = NF(\omega)\int_0^{2\pi} d\phi' \int_0^{\infty} r'dr'\rho(r')\tilde{E}(\Delta r,z,\omega) \tag{7}$$

where $(\Delta r)^2 = (r - r'\cos\phi')^2 + (r'\sin\phi')^2$. The longitudinal wake experienced by an electron traveling in the z-direction, that passes through the $z=0$ plane at time Δt, is

$$W(r,z,\Delta t) = E_N(r,z,t = z/\beta c + \Delta t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp[-i\omega(z/\beta c + \Delta t)]\tilde{E}_N(r,z,\omega)d\omega, \tag{8}$$

where β is the electron velocity divided by c. For an ultrarelativistic electron in the formation zone, the on-axis wake is therefore

$$W(r=0,z,\Delta t) = N\int_{-\infty}^{\infty} F(\omega)e^{-i\omega\Delta t}d\omega\int_0^{\infty} r'dr'\rho(r')\left[\left(\frac{-ie\omega}{2\pi\varepsilon_0 c^2\gamma^2}\right)K_0\left(\frac{\omega r'}{c\gamma}\right) - \left(\frac{e}{2\pi\varepsilon_0 c}\right)\frac{\exp\left(ik\sqrt{z^2 + r'^2} - ikz\right)}{\sqrt{z^2 + r'^2}}\right]. \tag{9}$$

The wake is the sum of a Coulomb wake and a coherent radiation wake, where the Coulomb wake is

$$W_{Coul}(0,z,\Delta t) = N\int_{-\infty}^{\infty} F(\omega)e^{-i\omega\Delta t}d\omega\int_0^{\infty} r'dr'\rho(r')\left[\left(\frac{-ie\omega}{2\pi\varepsilon_0 c^2\gamma^2}\right)K_0\left(\frac{\omega r'}{c\gamma}\right)\right] \tag{10}$$

and the coherent radiation wake is

$$W_{CR}(0,z,\Delta t) = N\int_{-\infty}^{\infty} F(\omega)e^{-i\omega\Delta t}d\omega\int_0^{\infty} r'dr'\rho(r')\left[\left(\frac{-e}{2\pi\varepsilon_0 c}\right)\frac{\exp\left(ik\sqrt{z^2 + r'^2} - ikz\right)}{\sqrt{z^2 + r'^2}}\right] \tag{11}$$

For a uniform density distribution within a beam radius of r_b [$\rho(r) = 1/\pi r_b^2$ for $r < r_b$], the wakes are

$$W_{Coul}(0,z,\Delta t) = \frac{-Ne}{2\pi^2\varepsilon_0 r_b^2}\int_{-\infty}^{\infty} F(\omega)e^{-i\omega\Delta t}\left(\frac{i}{\omega}\right)\left[1 - \frac{\omega r_b}{c\gamma}K_1\left(\frac{\omega r_b}{c\gamma}\right)\right]d\omega \tag{12}$$

and

$$W_{CR}(0,z,\Delta t) = \frac{-Ne}{2\pi^2\varepsilon_0 r_b^2}\int_{-\infty}^{\infty} F(\omega)e^{-i\omega\Delta t}\,\frac{\exp\left(ik\sqrt{z^2 + r_b^2} - ikz\right) - 1}{ick}\,d\omega \tag{13}$$

For $kr_b << 1$, the integrand of eq. (12) is much smaller than that of eq. (13) for $z << \lambda\gamma^2/[2\pi\ln(\lambda\gamma/2\pi r_b)]$. Therefore, when r_b is much smaller than the bunchlength $\sigma_z = c\sigma_t$, the coherent radiation wake dominates for $z << \sigma_z\gamma^2/\ln(\sigma_z\gamma/r_b)$.

For $r_b << \sigma_z$, the coherent radiation wake in the formation zone may be written in terms of the beam current profile $I(\Delta t) = Nef(\Delta t)$ as

$$W_{CR}(0,z,\Delta t) = f(\Delta t)\left(\frac{-Ne}{\pi\varepsilon_0 c r_b^2}\right)\left[\sqrt{z^2 + r_b^2} - z\right] = \frac{-I(\Delta t)}{\pi\varepsilon_0 c r_b^2}\left[\sqrt{z^2 + r_b^2} - z\right] \tag{14}$$

Since the wake is proportional to the beam current, it is resistive with resistance per unit length of $(\sqrt{z^2+r_b^2}-z)/(\pi\varepsilon_0 c r_b^2)$, which depends upon z. For $r_b << \sigma_z$ and $r_b << z$, eq. (14) becomes

$$W_{CR}(0,z,\Delta t) \approx \frac{-I(\Delta t)}{2\pi\varepsilon_0 cz} \tag{15}$$

For a bunch with $r_b \geq \sigma_z$, eq. (15) applies for $z >> r_b^2/\sigma_z$.

The coherent radiation wake results from radiation emitted during the bunch's acceleration. Thus, to calculate the coherent radiation wake of a bunch whose dimensions r_b and σ_z change as it propagates, one should use the dimensions of the bunch during the acceleration process.

5. Integrated wake

Because of the singular nature of the sudden-acceleration wake for $z \to 0, r_b \to 0$, our formation-zone wake expressions [eqs. (14) and (15)] should be applied only for values of z that are sufficiently large that the radiation field is described by that of a sudden acceleration. For linear acceleration over a distance d, this requires that $z >> d$ [2].

For an electron exiting a hard-edged bending magnet with radius of curvature ρ, the radiation in the formation zone is approximated by that of a sudden acceleration ("edge radiation") for $z >> d$, where $d = \rho(\lambda/z)^{1/2}$ is the distance required to deflect an electron through the angle $(\lambda/z)^{1/2}$ [2]. The radiation at wavelength λ is therefore approximated by that of a sudden acceleration when z exceeds the formation length of ordinary synchrotron radiation $\rho^{2/3}\lambda^{1/3}$. Thus, the coherent radiation wake of a bunch is approximated by our "edge radiation" expressions for $z >> d = \rho^{2/3}(2\pi\sigma_z)^{1/3}$.

In paraxial approximation, the forward transition radiation from a perfect conductor is equivalent to the radiation from a sudden acceleration. For a bunch passing through a realistic metal foil, the radiation emission occurs over a skin depth d. For a bunch exiting a plasma, the emission of forward transition radiation primarily occurs within the distance d over which the plasma's index of refraction changes from 0.8 to 1.0 [5]. For these cases, we expect our wake expressions to apply for $z >> d$.

When calculating the integrated wake, we restrict our integration to sufficiently large values of z that our wake expressions are approximately valid: $z > d$. Consequently, our calculations do not include the wake in the region where the bunch is accelerated.

When a bunch with $r_b << \sigma_z$ is accelerated over a distance $d << r_b$, the integrated on-axis wake as the bunch travels from axial coordinate d to axial coordinate z is

$$\begin{aligned}\Delta V(r=0,z,\Delta t) &= \int_d^z W_{CR}(0,z',\Delta t)dz' = \int_d^{\min(z,\sigma_z\gamma^2,h^2/2\pi\sigma_z)} \frac{-I(\Delta t)}{\pi\varepsilon_0 c r_b^2}\left[\sqrt{z'^2+r_b^2}-z'\right]dz' \\ &\approx \int_0^{\min(z,\sigma_z\gamma^2,h^2/2\pi\sigma_z)} \frac{-I(\Delta t)}{\pi\varepsilon_0 c r_b^2}\left[\sqrt{z'^2+r_b^2}-z'\right]dz' \approx \frac{-I(\Delta t)}{2\pi\varepsilon_0 c}\left\{\ln\left[\frac{\min(z,\sigma_z\gamma^2,h^2/2\pi\sigma_z)}{r_b}\right]+1.194\right\},\end{aligned} \tag{16}$$

where the approximate evaluation of the integral is accurate for $z > r_b$.

When a bunch with $r_b << \sigma_z$ is accelerated over a distance $d > r_b$, the integrated on-axis wake is

$$\Delta V(0,z,\Delta t)=\int_d^z W_{CR}(0,z',\Delta t)dz'=\int_d^{\min(z,\sigma_z\gamma^2,h^2/2\pi\sigma_z)}\frac{-I(\Delta t)}{2\pi\varepsilon_0 cz'}dz'=\frac{-I(\Delta t)}{2\pi\varepsilon_0 c}\ln\left[\frac{\min(z,\sigma_z\gamma^2,h^2/2\pi\sigma_z)}{d}\right] \quad (17)$$

When a bunch with $r_b \geq \sigma_z$ is accelerated over a distance d, the wake is approximated by eq. (15) for $z > \max(d, r_b^2/\sigma_z)$, giving an integrated wake of

$$\Delta V(0,z,\Delta t)\approx\int_{\max(d,r_b^2/\sigma_z)}^{\min(z,\sigma_z\gamma^2,h^2/2\pi\sigma_z)}\frac{-I(\Delta t)}{2\pi\varepsilon_0 cz'}dz'=\frac{-I(\Delta t)}{2\pi\varepsilon_0 c}\ln\left[\frac{\min(z,\sigma_z\gamma^2,h^2/2\pi\sigma_z)}{\max(d,r_b^2/\sigma_z)}\right] \quad (18)$$

When $d < r_b^2/\sigma_z$ eq. (18) does not include wake contributions from small values of z where eq. (15) does not apply: $d < z < r_b^2/\sigma_z$.

6. Edge radiation

Consider a bunch exiting a hard-edged bending magnet in which the orbit's radius of curvature is ρ. At a distance $z >> \rho^{2/3}(2\pi\sigma_z)^{1/3}$ from the magnet edge, the coherent radiation is primarily edge radiation described by our model. Therefore, eqs. (16)–(18) apply with $d \approx \rho^{2/3}(2\pi\sigma_z)^{1/3}$. The integrated on-axis edge-radiation wake for a bunch with $\rho^{2/3}(2\pi\sigma_z)^{1/3} >> \max(r_b, r_b^2/\sigma_z)$ is approximately

$$\Delta V(0,z,\Delta t)=\int_{\rho^{2/3}(2\pi\sigma_z)^{1/3}}^{z} W_{CR}(0,z',\Delta t)dz'=\int_{\rho^{2/3}(2\pi\sigma_z)^{1/3}}^{\min(z,\sigma_z\gamma^2,h^2/2\pi\sigma_z)}\frac{-I(\Delta t)}{2\pi\varepsilon_0 cz'}dz'=\frac{-I(\Delta t)}{2\pi\varepsilon_0 c}\ln\left[\frac{\min(z,\sigma_z\gamma^2,h^2/2\pi\sigma_z)}{\rho^{2/3}(2\pi\sigma_z)^{1/3}}\right] \quad (19)$$

Note that eq. (19) does not include contributions from the wake within the bending magnet or the wake within a distance $\rho^{2/3}(2\pi\sigma_z)^{1/3}$ of the magnet edge.

To test the validity of our model, we compared the wake given by eq. (15) with the computed wakes given in figs. (2) and (3) of Ref. [6] and fig. (3) of Ref. [7]. The computed wakes describe a 1 nC, 150 MeV Gaussian bunch with $\sigma_z = 50\ \mu\text{m}$ undergoing a $19°$ bend or $3.8°$ bend with radius of curvature ρ = 1.5 m. For $z > \rho^{2/3}(2\pi\sigma_z)^{1/3} = 89$ mm, good agreement is obtained. The approximate validity of the integrated wake was confirmed by comparing eq. (19) with fig. (4) of Ref. [6].

For a $19°$ bending magnet, fig. (4) of Ref. [6] shows that the integrated edge-radiation wake is smaller than the integrated wake within the bending magnet. However, fig. (4) also indicates that for a small bending angle of $3.8°$, the integrated wake of edge radiation dominates. Thus, eq. (19) may provide an approximation of the entire integrated wake for magnets that bend the beam by a few degrees, such as magnets in a bunch-compressor chicane. In a bunch compressor chicane, the bunch length is smallest in the final bending magnet, so that the integrated edge-radiation wake of the final bending magnet may approximate the integrated radiation wake of the entire chicane.

7. Forward transition radiation from laser-plasma acceleration

When an electron bunch is accelerated by laser-plasma acceleration, transition radiation is emitted when the bunch exits the plasma [5], while plasma screening prevents radiation from the bunch's acceleration from reaching the axis [8]. It is expected that the transition radiation is emitted over a distance of $d \sim 2.5\,\mu\text{m}$ [5]. Since forward transition radiation is equivalent to the radiation from a sudden acceleration, our model is expected to describe the coherent radiation wake for $z >> 2.5\,\mu\text{m}$.

As an application, consider the test-case table-top VUV FEL scenario described in Ref. [1], in which a bunch with $r_b \approx \sigma_z = 1\,\mu\text{m}$, peak current of 60 kA, and energy of 130 MeV is emitted from a plasma, after which the bunch radius expands. For this case, the radiation formation length is $\sigma_z \gamma^2 = 65$ mm. For $z > 65$ mm, we use eq. (18) to evaluate the integrated on-axis coherent radiation wake for the portion of the bunch where the current is 60 kA, obtaining a value of 37 MV when vacuum-chamber shielding is negligible. The energy of an electron in the center of a Gaussian bunch will be 37 MV less than the energy of an electron in the head or tail after traveling through the formation zone, for the case where their energies are equal when exiting the plasma. To decrease this energy variation by vacuum-chamber shielding within the formation zone requires a vacuum-chamber height less than $\sigma_z \gamma \sqrt{2\pi} = 0.6\,\text{mm}$. Additional energy variation will result from the effect of the Coulomb wake, whose evaluation is complicated by the change in bunch radius as the bunch propagates [1].

Now, consider the table-top xray FEL scenario described in Ref. [1], in which a bunch with $r_b \approx \sigma_z = 1\,\mu\text{m}$, peak current of 100 kA, and energy of 1.2 GeV is emitted from a plasma. Suppose that, after traveling a distance of 1 m, the bunch passes through a 3-m long undulator with small undulator parameter (so that the undulator radiation and longitudinal velocity change are negligible). The undulator is located within the radiation formation zone, whose length is $\sigma_z \gamma^2 = 5.5\,\text{m}$. From eq. (18), we find that the integrated on-axis coherent radiation wake at the undulator entrance is approximately 78 MV for the portion of the bunch where the current is 100 kA.

Equation (15) applies within the undulator, so that integrating the on-axis coherent radiation wake through the undulator gives

$$\Delta V(\Delta t) = \int_1^4 W_{CR}(0, z', \Delta t) dz' = \int_1^4 \frac{-I(\Delta t)}{2\pi\varepsilon_0 c z'} dz' = \frac{-I(\Delta t)}{2\pi\varepsilon_0 c} \ln(4) \tag{20}$$

For the portion of the bunch where the current is 100 kA, eq. (20) gives an energy loss of 8.3 MV over the 3-m undulator length. The energy loss within the undulator may be prevented if the vacuum chamber shields the wake for $z > 1\,\text{m}$, which requires a chamber height obeying $h^2 / 2\pi\sigma_z < 1\,\text{m}$, i.e. $h < 2.5\,\text{mm}$. Additional energy variation will result from the Coulomb wake [1].

For a large undulator parameter, the bunch's longitudinal velocity in the undulator is decreased so that the wake moves faster than the bunch. In addition, the wake from transition undulator radiation emitted at the undulator entrance will influence the bunch [9]. Therefore, eq. (20) is not expected to accurately model the wake in an undulator with a large undulator parameter.

The wakes calculated above, which equal several MV, are sufficiently large that they may spoil the FEL process [1]. Thus, it is important to consider the effect of the coherent-radiation wake in addition to the Coulomb wake.

8. Conclusion

We have calculated the wake of a suddenly accelerated electron bunch, which also describes the wake of forward transition radiation. This wake also approximates the "edge-radiation" wake of a bunch exiting a bending magnet at distances from the magnet exceeding the formation length of the magnet's coherent synchrotron radiation.

A comparison with the computed wake downstream of a bending magnet yields good agreement. We also consider the wake for schemes in which a bunch produced by laser-plasma acceleration drives a VUV or xray FEL. In these cases, the large integrated wakes (several MV) may spoil the FEL process.

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[1] G. Geloni, E. Saldin, E. Schneidmiller and M. Yurkov, DESY 06-222 (2006).

[2] R. A. Bosch and O. V. Chubar, in *Synchrotron Radiation Instrumentation*, edited by E. Fontes, AIP Conf. Proc. No. 417 (AIP, New York, 1997), p. 35.

[3] R. A. Bosch, Nucl. Instrum. Methods Phys. Res., Sect. A 482, 789 (2002).

[4] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed.

[5] J. van Tilborg, C. B. Schroeder, C. V. Filip, Cs. Tóth, C. G. R. Geddes, G. Fubiani, E. Esarey and W. P. Leemans, Phys. Plasmas 13, 056704 (2006).

[6] G. Stupakov and P. Emma, in *Proceedings of the Eighth European Particle Accelerator Conference, Paris* (EPS-IGA/CERN, Geneva, 2002), p. 1479.

[7] M. Dohlus and T. Limberg, Nucl. Instrum. Methods Phys. Res., Sect. A 393, 494 (1997).

[8] R. A. Bosch, Phys. Plasmas 13, 113107 (2006).

[9] K. J. Kim, Phys. Rev. Lett. 76, 1244 (1996).