The use of a 1500-MHz third-harmonic cavity at the ALBA 3-GeV electron storage ring is studied. Analytic modeling and simulations indicate that such a cavity can successfully suppress parasitic coupled bunch instabilities and lengthen the bunch. The microwave instability is not expected for the estimated broadband impedance of ALBA. To achieve optimal bunch lengthening without exciting a fast longitudinal instability, a nonzero rf coupling coefficient for the harmonic cavity may be required.
1. Introduction

The ALBA 3-GeV ring being constructed in Barcelona, Spain will utilize a 500-MHz rf system with 6 fundamental rf cavities. A 1500-MHz passive superconducting harmonic cavity may be used to lengthen the bunch, thereby suppressing parasitic longitudinal coupled bunch instabilities and increasing the Touschek lifetime. In this technical note, we describe analytic modeling and simulations that predict instabilities for harmonic-cavity operation. The methods and notation are described in Refs. [1] and [2].

2. ALBA modeling

Before studying the effect of a harmonic cavity upon parasitic longitudinal coupled bunch instabilities, we use analytic modeling and macroparticle simulations to examine the coupled dipole and quadrupole Robinson instabilities that may be excited by tuning in the harmonic cavity [1]. These instabilities may prevent “optimal bunch lengthening,” where a long bunch is confined in a in an effective potential whose linear synchrotron frequency is zero.

The natural bunch length without a harmonic cavity is calculated to be 15.4 ps, while using the harmonic cavity to achieve optimal lengthening (where the linear synchrotron frequency is zero) is calculated to give a bunchlength of 56 ps for ring currents of 100–500 mA. The bunch is therefore lengthened by a factor of 3.6, provided that optimal bunch lengthening is feasible.

In fig. 1(a), we show analytic Robinson-instability predictions for ALBA when all rf buckets are full. A coupled-quadrupole Robinson instability is predicted for optimally lengthened and overstretched bunches when the ring current exceeds 300 mA. For tuning angles exceeding $\approx 89.98^\circ$, the algorithm used to compute the coupled-dipole and coupled-quadrupole Robinson frequencies does not converge, in which case a “W” is plotted. In many of these cases, the algorithm used to compute the dipole Robinson frequency without consideration of coupling with the quadrupole mode also does not converge. Non-convergence could be a result of code deficiencies. Alternatively, it may indicate that the assumption of slowly growing or damped Robinson oscillations is violated, which is consistent with a fast instability. In Appendix A we discuss a possible fast instability. It appears that this fast instability also describes a growing dipole Robinson mode after its growth rate has increased to a value larger than its oscillation frequency, in which case the instability should already be predicted by our code.

Figure 1(b) shows simulation results when 10 macroparticles/bucket are simulated for 200,000 turns. If a macroparticle’s displacement from the synchronous phase exceeds 1 ns (one-half the rf period), it is considered lost, in which case it is no longer tracked and its contribution to the wake is no longer computed. Nearly all the macroparticles are lost when the harmonic cavity is tuned in past a threshold tuning angle that depends upon ring current. The macroparticles are lost from the rf bucket within several thousand turns, preventing optimal bunchlengthening from being obtained. Consequently, the bunch can only be lengthened by a factor of ~2.

In simulations, losses within ~2000 turns occur for the approximate parameters where the analytic code cannot converge on coupled-dipole and -quadrupole Robinson frequencies and damping/growth rates, or coupled-quadrupole Robinson instability is predicted. After a single or a few growing oscillations about the synchronous phase, the bunch arrival time becomes increasingly negative until particles escape the bucket. Near threshold, they are lost after ~2000 turns, versus ~120 turns when the “fast instability” is simulated by neglecting the fundamental rf cavity or synchrotron radiation.

The rapid loss of macroparticles is illustrated in fig. 2(a) for a current of 33 mA and harmonic-cavity tuning angle of $-89.964^\circ$; where 100 macroparticles/bucket were modeled. For higher currents of 100
mA and 167 mA, figs. 2(b) and 2(c) show that the losses are more rapid. The losses in fig. 2 occur for parameters where the Robinson computation does not converge.

In fig. 3(a), analytic predictions also consider the possibility of worst-case parasitic coupled bunch instability, where the HOM resonant frequency is an integral multiple of the revolution frequency. We consider an HOM resonant frequency that equals 606 times the revolution frequency with impedance of 64.8 kΩ and quality factor of 310. The predictions of parasitic coupled bunch instability are in approximate agreement with the simulations shown in fig. 3(b). For high ring currents, tuning in the harmonic cavity suppresses the parasitic coupled bunch instability, but macroparticles are lost in simulations if the cavity is tuned in too far.

In fig. 4, we consider the same harmonic cavity with an rf coupling coefficient $\beta_2 = 3$. This reduces both the quality factor $Q_2$ and resonant impedance $R_2$ by a factor of 4. Consequently, the cavity must be tuned in further to obtain optimal bunch lengthening. In simulations, stable bunches with optimal bunch lengthening ($\sigma_L = 56$ ps) are observed for currents of 433–500 mA. In this current range, the bunches can be further lengthened to obtained stable double-hump bunches with lengths up to 80 ps.

In fig. 5, we consider the use of two harmonic cavities to increase the ratio $R_2 / Q_2$ by a factor of two. Stable bunches with optimal bunch lengthening are observed for current of 300–500 mA.

The excitation of microwave instability by broadband impedance is considered in fig. 6, for the case of a single third-harmonic cavity with an rf coupling coefficient $\beta_2 = 0$. A broadband impedance with $|Z_p / p| = 10 \Omega$ is modeled by a HOM with resonant angular frequency $\omega_c = c / b = 2.1429 \times 10^{10} \text{rad/s}$, where $c$ is the speed of light and $b = 14 \text{mm}$ is the half-height of the vacuum chamber. The quality factor $Q_3$ is one and the resonant impedance is $R_3 = 30560 \Omega$. Figure 6(a) shows microwave-instability predictions by the Boussard criterion [2, 3] while fig. 6(b) shows the results from simulations of 4000 macroparticles/bucket performed by the University of Wisconsin-Madison’s CONDOR® pool for high-throughput computing [4]. In the simulations, increased energy spread indicating instability is observed for ring currents exceeding $\sim 1.5$ times the threshold current predicted by the Boussard criterion, similar to previous studies of the microwave instability [2]. When the harmonic cavity tuning angle is $< -89.997^\circ$ for sufficiently large currents, macroparticles are lost or the bunch centroid is shifted by more than 2.35$\sigma_L$ from its initial synchronous phase, which may indicate the presence of a large potential well distortion or equilibrium phase instability [5].

The microwave instability is not observed in simulations for broadband impedance $\leq 5 \Omega$ and it is not predicted analytically for broadband impedance $< 3 \Omega$. Since the expected broadband impedance of ALBA is $\sim 0.45 \Omega$, occurrence of the microwave instability is not expected.

3. Summary

Analytic modeling and simulations suggest that a harmonic cavity may be successfully used to lengthen the bunch and suppress parasitic coupled bunch instability at ALBA. According to simulations, achieving optimal bunch lengthening without exciting a fast longitudinal instability may require a nonzero rf coupling coefficient of the harmonic cavity, e.g. an rf coupling coefficient $\beta_2 = 3$. 

3
Appendix A: a fast longitudinal instability

The loss of macroparticles within several synchrotron-oscillation periods suggests the occurrence of a “fast” or “strong” instability, whose rise time is smaller than the synchrotron oscillation period. Since the coupled-dipole Robinson frequency does not approach zero at the threshold of macroparticle losses, the “equilibrium phase” or “2nd Robinson” instability, which is a fast dipole Robinson instability, is not responsible.

We consider instead the positive feedback when bunches lose energy to the passive cavity, thereby increasing their revolution frequency. With an increased revolution frequency, the bunches excite the cavity at a frequency closer to resonance, and therefore lose energy to the cavity even faster, giving positive feedback. Assuming that the instability is fast, we neglect the energy gain from the fundamental rf cavity and energy loss from the emission of synchrotron radiation.

Let \( \omega(t) \) denote the bunch revolution frequency as a function of time, and

\[
\phi_2(t) = \tan^{-1}\left[ Q_2 (\nu M \omega(t) - \omega_2) (\nu M \omega(t) + \omega_2) / \omega_2 \nu M \omega(t) \right] = \tan^{-1}\left[ 2Q_2 (\nu M \omega(t) - \omega_2) / \omega_2 \right] < 0
\]

denote the time-dependent tuning angle that describes its interaction with the harmonic cavity. Let \( \Delta \omega(t) = \omega(t) - \omega_2 / \nu M < 0 \) denote the difference between the time-dependent revolution frequency and the frequency where resonant excitation of the harmonic cavity will occur. We assume that \( \omega(t) \) changes sufficiently slowly so that the cavity excitation at time \( t \) equals the steady-state excitation at frequency \( \omega(t) \). We neglect any variation of the bunch form factor.

For revolution frequency \( \omega \), the impedance from the harmonic cavity is \( R_2 \cos(\phi_2) \exp(i\phi_2) \), with real part \( R_2 \cos^2(\phi_2) \). For tuning angles near to \(-90^\circ\), the real part is approximately \( R_2 / \tan^2(\phi_2) \). The average energy of an electron decreases from excitation of the harmonic cavity according to

\[
de \epsilon / dt = -2eIR_2 F_2^2 \cos^2(\phi_2) / T_0 \tag{A1}
\]

For a positive momentum compaction \( \alpha \) and \( \omega(t) = \omega_0 \), where \( \omega_0 \) is the initial revolution frequency, the revolution frequency changes according to

\[
\frac{d\omega}{dt} = \frac{-\alpha d\epsilon / dt}{E_0} \tag{A2}
\]

Therefore,

\[
\frac{d\omega}{dt} = \frac{-\alpha \omega_0 d\epsilon / dt}{E_0 T_0} = \frac{2\alpha \omega_0 eIR_2 F_2^2 \cos^2(\phi_2)}{E_0 T_0} = \frac{2\alpha \omega_0 eIR_2 F_2^2}{E_0 T_0 \tan^2(\phi_2)} = \frac{\alpha \omega_0^3 eIR_2 F_2^2}{2E_0 T_0 Q_2^2 (\Delta \omega(t))^2} \tag{A3}
\]

so that

\[
\Delta \omega^3 \frac{d(\Delta \omega)}{dt} = \frac{1}{3} \frac{d(\Delta \omega)^3}{dt} = \frac{\alpha \omega_0^3 eIR_2 F_2^2}{2E_0 T_0 Q_2^2} \tag{A4}
\]

Equation (A4) has solution

\[
[\Delta \omega(t)]^3 = [\Delta \omega(0)]^3 + \frac{3\alpha \omega_0^3 eIR_2 F_2^2}{2E_0 T_0 Q_2^2} t \tag{A5}
\]

According to eq. (A5), the revolution frequency increases at an accelerating rate, so that it resonantly excites the harmonic cavity after the “rise-time”

\[
\tau = \frac{-2E_0 T_0 Q_2^2 [\Delta \omega(0)]^3}{3\alpha \omega_0^3 eIR_2 F_2^2} = \frac{-E_0 T_0}{12\alpha Q_2^2 eIR_2 F_2^2} \tan^3(\phi_2) \approx \frac{E_0 T_0}{12\alpha Q_2^2 eIR_2 F_2^2 \cos^3(\phi_2)} \tag{A6}
\]

where \( \phi_2 \) is the tuning angle at time \( t = 0 \).
In order that this fast-instability solution be valid, the time $\tau$ must be smaller than the synchrotron oscillation period in the absence of the harmonic cavity

$$\tau < \frac{2\pi}{\omega},$$  \hspace{1cm} (A7)

In addition, the time $\tau$ must be sufficiently long so that the cavity field at time $t$ equals the steady-state value when driven at frequency $\omega(t)$, which is similar to the requirement for accurate measurement from a swept-frequency spectrum analyzer. Letting $\Delta\omega = \omega_2 - \nu M_0$ equal the effective bandwidth of the harmonic cavity’s response at time $t = 0$, this requires that $Mv\left(\frac{d\omega}{dt}\right) < \Delta\omega^2$, where $Mv\left(\frac{d\omega}{dt}\right) \approx \Delta\omega / \tau$. Thus, we obtain the additional requirement for our solution that

$$\tau > \frac{1}{\Delta\omega} = \frac{2Q_2}{\omega_2 \tan(\phi_2)}$$ \hspace{1cm} (A8)

If eq. (A8) is not satisfied, we expect that the instability rise time may increase to a value of $\sim 2Q_2 / \omega_2 \tan(\phi_2)$. Because the cavity is excited off-resonance, this value is much smaller than $Q_2$ times the cavity’s natural period, which equals $2\pi Q_2 / \omega_2$.

The rate $1/\tau$ given by eq. (A6) is three times the dipole Robinson growth rate contribution from the harmonic cavity impedance, suggesting that our model of fast instability may describe the growth of a slowly growing dipole Robinson instability after its initial growth. In this case, the instability should already be predicted by our code.

Alternatively, our model of a fast dipole Robinson instability may approximate the fast loss of macroparticles after some unstable bunch oscillations have taken place.

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<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
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<tr>
<td>Ring energy</td>
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<tr>
<td>Fundamental cavity load angle</td>
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Table I. Parameters for parasitic longitudinal coupled bunch instability at ALBA (courtesy of T. Günzel).
Figure 1. Instability modeling for passive harmonic-cavity operation at ALBA. A solid curve shows the parameters for optimal bunch lengthening. To the left of this curve, the bunches are “understretched” with an approximately Gaussian profile. To the right, the bunches are “overstretched” double-hump bunches. (a) Analytic instability predictions. |: coupled dipole Robinson instability; *: coupled quadrupole Robinson instability; #: fast mode-coupling Robinson instability; c: coupled-bunch instability with longitudinal mode number of 1; W: coupled dipole-quadrupole Robinson computation does not converge. (b) Results of 200,000-turn simulations of 4480 macroparticles. •: mild instability, where the energy spread exceeds its natural value by 10–30%; ○: moderate instability, where the energy spread has increased by 30–100%; ●: strong instability, where the energy spread has increased more than 100%; ■: lost macroparticles.
Figure 2. Bunch centroid position (tbar) and rms bunch length (sigmat) as macroparticles are lost at the beginning of simulations of 44,800 macroparticles. If a macroparticle’s displacement from the synchronous phase exceeds 1 ns (one-half the rf period), it is considered lost, in which case it is no longer tracked and its contribution to the wake is no longer computed. (a) Ring current of 33 mA and harmonic cavity tuning angle of -89.96416 degrees. (b) Ring current of 100 mA and harmonic cavity tuning angle of -89.96416 degrees. (c) Ring current of 167 mA and harmonic cavity tuning angle of -89.96416 degrees.
Figure 3. Instability modeling for passive harmonic-cavity operation at ALBA, in which worst-case parasitic coupled bunch instability is considered. (a) Analytic instability predictions. -: parasitic coupled bunch instability, |: coupled dipole Robinson instability; *: coupled quadrupole Robinson instability; #: fast mode-coupling Robinson instability; c: coupled-bunch instability with longitudinal mode number of 1; W: coupled dipole-quadrupole Robinson computation does not converge. (b) Results of 200,000-turn simulations of 4480 macroparticles. o: mild instability, where the energy spread exceeds its natural value by 10–30%; •: moderate instability, where the energy spread has increased by 30–100%; O: strong instability, where the energy spread has increased more than 100%; ■: lost macroparticles.
Figure 4. Instability modeling for passive harmonic-cavity operation at ALBA when the harmonic cavity has rf-coupling coefficient of 3. Worst-case parasitic coupled bunch instability is considered. (a) Analytic instability predictions. -: parasitic coupled bunch instability, |: coupled dipole Robinson instability; *: coupled quadrupole Robinson instability; #: fast mode-coupling Robinson instability; c: coupled-bunch instability with longitudinal mode number of 1; W: coupled dipole-quadrupole Robinson computation does not converge. (b) Results of 200,000-turn simulations of 4480 macroparticles. o: mild instability, where the energy spread exceeds its natural value by 10–30%; o: moderate instability, where the energy spread has increased by 30–100%; O: strong instability, where the energy spread has increased more than 100%; ■: lost macroparticles.
Figure 5. Instability modeling for passive harmonic-cavity operation at ALBA when two identical harmonic cavities are used. Worst-case parasitic coupled bunch instability is considered. (a) Analytic instability predictions. -: parasitic coupled bunch instability, |: coupled dipole Robinson instability; *: coupled quadrupole Robinson instability; #: fast mode-coupling Robinson instability; c: coupled-bunch instability with longitudinal mode number of 1; W: coupled dipole-quadrupole Robinson computation does not converge. (b) Results of 200,000-turn simulations of 4480 macroparticles. o: mild instability, where the energy spread exceeds its natural value by 10–30%; o: moderate instability, where the energy spread has increased by 30–100%; o: strong instability, where the energy spread has increased more than 100%; ■: lost macroparticles.
Figure 6. Instability modeling for single passive harmonic-cavity operation at ALBA with an HOM representing a broadband impedance $|Z_p/p| = 10 \Omega$. (a) Analytic instability predictions. m: microwave instability, |: coupled dipole Robinson instability; *: coupled quadrupole Robinson instability; #: fast mode-coupling Robinson instability; c: coupled-bunch instability with longitudinal mode number of 1; W: coupled dipole-quadrupole Robinson computation does not converge. (b) Results of 200,000-turn simulations of 1,792,000 macroparticles. o: mild instability, where the energy spread exceeds its natural value by 10–30%; o: moderate instability, where the energy spread has increased by 30–100%; O: strong instability, where the energy spread has increased more than 100%; ■: lost macroparticles; ×: energy spread is within 10% of its natural value while the bunch centroid is shifted by more than 2.35 $\sigma$, from its initial synchronous phase.