From angle resolved photoemission (ARPES) experiments on Bi(111) in the vicinity of the Fermi level we were able to extract the contributions of the electron-phonon interaction to the band structure.\(^1\) Contrary to many other metals the Debye temperature of bismuth is small ($\theta_D = 120$ K) so that the effect of the ep-interaction will be limited to an energy region of about 10meV below the Fermi level.

Fig. 1 shows the two dimensional (2D) band structure of Bi(111) in the vicinity of the $\Gamma$ -point of the surface Brillouin zone close to the Fermi level. We have extracted the band dispersion $E(k_{||})$ by fitting the peak positions of a data set containing 900 energy distribution curves. The band (blue mesh) generating the electron pocket in the center as well as the band (red mesh) generating the six hole pockets of the 2D Fermi surface are shown. The analysis of the ep-interaction was performed on the hole band (red mesh) along the $k_x$ -axis.

Assuming that the photoemission matrix element is independent of $k$ and a slowly varying function of the initial state energy, ARPES measures the hole spectral distribution function $A(\omega)$.\(^2\) It is convenient to model the interactions causing non-quasiparticle behavior by using the complex one electron self-energy $\Sigma(\omega) = \text{Re} \Sigma(\omega) + i \text{Im} \Sigma(\omega)$. The spectral function can then be written as:
\[ A(\omega) = \frac{\pi^{-1}|\text{Im} \Sigma(\omega)|}{[\omega - \varepsilon(k) - \text{Re} \Sigma(\omega)]^2 + |\text{Im} \Sigma(\omega)|^2} \]

For a constant energy \( \omega \) the single particle spectral function \( A(\omega,k) \) has a Lorentzian line shape with a full width at half maximum of \( \Gamma = 2|\text{Im} \Sigma(\omega)| \). Due to the self-energy contribution the energy position of the photoemission peak is at \( E(k) = \varepsilon(k) + \text{Re} \Sigma(\omega) \) where \( \varepsilon(k) \) is the bare dispersion in the one-electron band structure. What is measured in ARPES is the experimental band structure \( E(k) \) which is the sum of the bare dispersion and the self-energy contribution. In special cases one can directly extract the real part of the self-energy by linearly extrapolating the measured band dispersion towards the Fermi level.\(^3\) The linear extrapolation is not possible for Bi(111) because of its highly anisotropic band structure. An unambiguous, direct determination of the real part of the self-energy using the relation \( \text{Re} \Sigma(\omega) = E(k) - \varepsilon(k) \) is thus not possible. However, keeping in mind that \( \text{Re} \Sigma(\omega) \) and \( \text{Im} \Sigma(\omega) \) are related to each other by means of a dispersion relation, it is possible to extract \( \text{Re} \Sigma(\omega) \) through a Hilbert transform from \( \text{Im} \Sigma(\omega) \) which one can obtain from the line widths by fitting the spectral function \( A(\omega,k) \) to momentum distribution curves (MDC).

Figure 2

Figure 2(a) shows the result of MDC line shape fits where \( \text{Im} \Sigma(\omega) \) was a free fit parameter while the peak position given by \( E(k) = \varepsilon(k) + \text{Re} \Sigma(\omega) \) was an input
parameter obtained from the experimental dispersion $E(k)$. Over most of the spectral range $\text{Im} \Sigma(\omega)$ is constant at 32meV and drops off closer to the Fermi level to 28meV. The individual error bars in the figure are $\pm \sigma$ standard deviation. The energy dependence of $\text{Im} \Sigma(\omega)$ shows the characteristics of the electron-phonon interaction in the Debye model. To verify that ep-coupling can cause the observed minute $\omega$-dependence in $\text{Im} \Sigma(\omega)$, we fitted our data to the following electron-phonon scattering amplitude:

$$\text{Im} \Sigma(\omega) = \pi \int \lambda \left( \frac{\omega'}{\omega_D} \right)^2 \left[ 1 - f(\omega' - \omega, T) + 2n(\omega', T) + f(\omega' + \omega, T) \right] d\omega' + \Sigma_0$$

The first part of the equation is based on the Debye model where the strength of the ep-interaction is represented by the electron-phonon coupling constant $\lambda$.$^4$ Differences in the electron and phonon statistics are considered through the Fermi-Dirac distribution function $f(\omega, T)$ and the Bose-Einstein distribution function $n(\omega, T)$. The constant part $\Sigma_0$ attributes to scattering processes involving impurities and surface roughness. The solid line is the result of a free fit to the ep-scattering amplitude. Using the Debye energy of $\omega_D = 10.3\text{meV}$ and a sample temperature of 50K as an input parameter we fitted the electron-phonon coupling constant $\lambda = 0.6 \pm 0.05$. The constant part $\Sigma_0$ was fitted to 22meV resulting in a finite temperature contribution of 6meV from the ep-interaction at the Fermi level.

Applying the Hilbert transform we calculated the real part of the self-energy from the data points shown in Fig. 2(a). The calculated values of $\text{Re} \Sigma(\omega)$ are shown in Fig. 2(b). For comparison we also show as a solid line the Hilbert transforms of the $\text{Im} \Sigma(\omega)$ function. As one can see from Fig. 2(b) the mass renormalization effects are limited to an energy range very close to the Fermi level are small and are affecting the experimental dispersion only in a range of about 20meV below the Fermi level. To our knowledge this the smallest reported energy scale for the ep-interaction determined by photoemission spectroscopy.

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